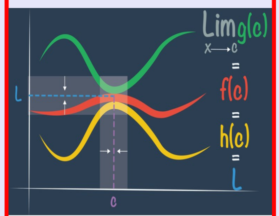
Calculus I

Lecture 29



Feb 19-8:47 AM

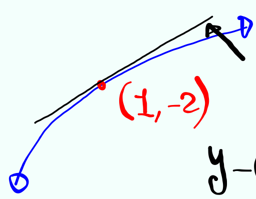
Class Quiz 13

Find equation of the tan. line to the graph of $y^2 = \sqrt{x} + 3$ at $(1,-2)$.

Final Ans. in Slope-Int. Form

$(-2)^2 = \sqrt{1} + 3$

$4 = 4$ ✓



$m = \left.\frac{dy}{dx}\right|_{(1,-2)}$

$2y\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$2(-2)\,m = \frac{1}{2\sqrt{1}}$

$-4m = \frac{1}{2}$

$\boxed{m = \frac{-1}{8}}$

$y - (-2) = \frac{-1}{8}(x - 1)$

$y + 2 = \frac{-1}{8}x + \frac{1}{8}$

$\boxed{y = \frac{-1}{8}x - \frac{15}{8}}$

Oct 17-6:56 AM

Find $\frac{dy}{dx}$

$$\underbrace{x^2 \cos(x^2+y^2)}_{\text{Product}} = 1$$

$$\frac{d}{dx}[x^2]\cdot\cos(x^2+y^2) + x^2\cdot\frac{d}{dx}\left[\cos(x^2+y^2)\right] = 0$$

$$2x\cos(x^2+y^2) + \boxed{x^2\cdot -\sin(x^2+y^2)}\cdot\left(2x+2y\frac{dy}{dx}\right) = 0$$

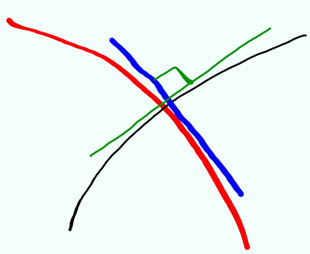
$$2x\cos(x^2+y^2) - 2x^3\sin(x^2+y^2) - 2yx^2\sin(x^2+y^2)\frac{dy}{dx} = 0$$

$$2x\cos(x^2+y^2) - 2x^3\sin(x^2+y^2) = 2yx^2\sin(x^2+y^2)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x\cos(x^2+y^2) - 2x^3\sin(x^2+y^2)}{2yx^2\sin(x^2+y^2)}$$

Oct 17-7:40 AM

Show $\boxed{y = cx^2}$ and $x^2+2y^2 = K$ are orthogonal curves.



$$\frac{d}{dx}y = \frac{d}{dx}[cx^2] \qquad 2x + 4y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 2cx \qquad \frac{dy}{dx} = \frac{-2x}{4y}$$

Product of slopes $= -1$

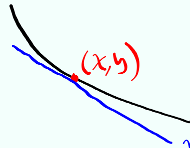
Product of derivatives $= -1$

$$2cx\cdot\frac{-2x}{4y} = \frac{-\cancel{4}cx^2}{\cancel{4}y} = -\frac{cx^2}{y} = -\frac{y}{y} = \boxed{-1}$$

Oct 17-7:47 AM

Find all points on the curve $x^2y^2 + xy = 2$ where slope of the tan. line is -1.

$$2xy^2 + x^2 \cdot 2y\frac{dy}{dx} + 1 \cdot y + x \cdot \frac{dy}{dx} = 0$$



$$2xy^2 - 2yx^2 + y - x = 0$$

$m = -1 \rightarrow \left.\frac{dy}{dx}\right|_{(x,y)} = -1$

$$\begin{cases} x^2y^2 + xy = 2 \\ \underbrace{2xy^2 - 2yx^2}\ \underbrace{+y-x} = 0 \end{cases}$$

$$2xy(y-x) + 1(y-x) = 0$$

$$(y-x)(2xy+1) = 0$$

$y - x = 0$ $\quad$ $2xy + 1 = 0$

$\boxed{y = x}$ $\quad$ $y = \frac{-1}{2x}$

$x^2y^2 + xy = 2$

when $y = x$

$x^2x^2 + xx = 2$

$x^4 + x^2 - 2 = 0$

$(x^2+2)(x^2-1) = 0$

x^2+2: has no real solutions $\quad$ $x^2 - 1 = 0$, $x = \pm 1$

$\boxed{\begin{matrix}(1,1) \\ (-1,-1)\end{matrix}}$

Oct 17-7:52 AM

$$x^2y^2 + xy = 2$$

when $y = \frac{-1}{2x}$

$$\cancel{x^2} \cdot \frac{1}{4\cancel{x^2}} + \cancel{x} \cdot \frac{-1}{2\cancel{x}} = 2$$

$$\frac{1}{4} - \frac{1}{2} \neq 2$$

No Solutions

(1, 1)

(−1, −1)

Oct 17-8:02 AM

Use Quadratic approximation to estimate $\sqrt{10}$

$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$

by Calc

$\sqrt{10} \approx \boxed{3.162}27766$

$f(x) = \sqrt{x}$

$a = 9$

$f(9) = \sqrt{9} = 3$

$f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$

$f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

$f''(x) = \frac{1}{2} \cdot \frac{-1}{2} x^{-3/2} = \frac{-1}{4x\sqrt{x}}$

$f''(9) = \frac{-1}{4 \cdot 9 \cdot \sqrt{9}} = \frac{-1}{108}$

$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$

$\sqrt{x} \approx 3 + \frac{1}{6}(x-9) - \frac{1}{216}(x-9)^2$

Replace x with 10

$\sqrt{10} \approx 3 + \frac{1}{6} \cdot 1 - \frac{1}{216} \cdot 1 = 3 + \frac{1}{6} - \frac{1}{216}$

$= \frac{683}{216}$

$\approx \boxed{3.162}037037$

Oct 17-8:05 AM

Use Quadratic approximation to estimate

$\tan 46^\circ \approx \tan 45^\circ = 1$

$f(x) = \tan x$

$a = 45^\circ$

$f(45^\circ) = \tan 45^\circ = 1$

$f'(x) = \sec^2 x$

$f'(45^\circ) = \sec^2 45^\circ = (\sqrt{2})^2 = 2$

$f''(x) = 2 \sec x \cdot \sec x \tan x$

$f''(45^\circ) = 2 \cdot \sec^2 45^\circ \tan 45^\circ$
$= 2 \cdot (\sqrt{2})^2 \cdot 1 = 4$

$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$

$\tan x \approx 1 + 2\left(x - \frac{\pi}{4}\right) + \frac{4}{2}\left(x - \frac{\pi}{4}\right)^2$

$\tan x \approx 1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2$

Replace x with 46° (Rad)

$\tan 46^\circ \approx 1 + 2 \cdot 1^\circ + 2 \cdot 1^{\circ 2}$

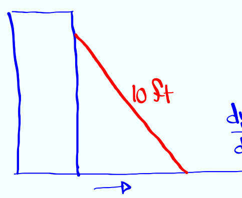
$= 1 + 2 \cdot \frac{\pi}{180} + 2 \cdot \left(\frac{\pi}{180}\right)^2$

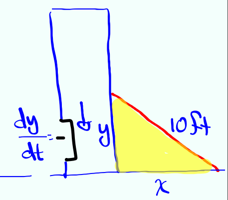
$\tan 46^\circ \approx \boxed{1.0355}1582$

Calc. directly $\approx \boxed{1.0355}30314$

Oct 17-8:13 AM

A ladder is 10-ft long.
It is leaning against a wall.
The bottom of the ladder pulls away from the wall at 2ft/min.
How fast is the top of the ladder sliding down when bottom is 6 ft from the wall?





$\frac{dx}{dt} = 2$ ft/min

$x^2 + y^2 = 10^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$6^2 + y^2 = 10^2$

$y = 8$

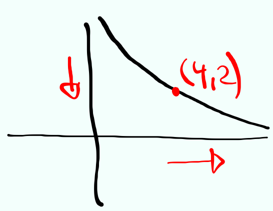
$6 \cdot 2 + 8 \cdot \frac{dy}{dt} = 0$

$8\frac{dy}{dt} = -12$

$\frac{dy}{dt} = \frac{-12}{8} = -1.5$ ft/min

Oct 17-8:23 AM

An object is moving along the curve
$xy = 8$
y-coordinate is decreasing at 3cm/s.
How fast is x-coordinate changing at (4,2)?



$\frac{d}{dt}[xy] = \frac{d}{dt}[8]$

$\frac{dx}{dt} \cdot y + x \cdot \frac{dy}{dt} = 0$

$\frac{dx}{dt} \cdot 2 + 4 \cdot (-3) = 0$

$\frac{dx}{dt} = \frac{12}{2} = 6$ cm/s

+

Oct 17-8:30 AM